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Preservation by Homomorphisms and Infinitary Languages

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Abstract In this paper we study when sentences of infinitary languages are preserved by homomorphisms. This is done by using generalized Henkin construction. By the same technique we can also study when a sentence has an equivalent sentence which is in normal form.

The so-called Hintikka game, which is a straightforward generalization of Henkin construction, is used in, e.g., Hyttinen [1], [2] and Oikkonen [6]. In this paper we refine the game and prove a preservation theorem by using it.

Throughout this paper we assume that κ is weakly compact. This is done because in the proofs we construct certain trees, which have no branches of length $\geq \kappa$ and no nodes that have $\geq \kappa$ immediate successors, and then under the assumption that κ is weakly compact we know that the trees are of cardinality $< \kappa$.

We begin this chapter by defining the language $M_{\kappa\kappa}$. This language was first defined and studied by M. Karttunen in [4]. By a λ κ-tree T we mean a tree such that each node has $< \lambda$ immediate successors, there are no branches of length $\geq \kappa$ and if x and y are limit nodes and $\{t \in T \mid t < x\} = \{t \in T \mid t < y\}$ then $x = y$.

1.1 Definition $\phi = (T, l)$ is a formula of $M_{\kappa\kappa}$ if

(1) T is a $\kappa\kappa$-tree without branches of limit length, i.e. every branch has a maximal element;
(2) l is a labeling function with the properties
  a: if $t \in T$ does not have any successors then $l(t)$ is either an atomic or negated atomic formula;
  b: if $t \in T$ has exactly one immediate successor then $l(t)$ is of the form $\exists x$ or $\forall x$, x variable;
  c: if $t \in T$ has more than one immediate successor then $l(t)$ is either $\vee$ or $\wedge$.

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