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Stability for Pairs of Equivalence Relations

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Abstract We consider pairs of equivalence relations E_0, E_1 such that, for some nonnegative integer h, every class of the join of E_0 and E_1 contains at most h classes of either E_0 or E_1. We classify these structures under categoricity (in some infinite power), nonmultidimensionality and finite cover property.

1 Let T be a countable complete first-order theory with no finite models. As usual, we assume that all models of T are elementary substructures of some big model U (the universe of T). Our aim is to study stability for theories T of two equivalence relations E_0, E_1, with particular attention to the problem of classifying among them the ones that are categorical in $\aleph_0$ or in $\aleph_1$.

Notice that in the simple case $E_0 = E_1$, hence when there is a unique equivalence relation, the situation is quite clear. In fact T is ω-stable and one can easily prove:

Theorem 1 *Let T be the theory of an equivalence relation E. Then the following propositions are equivalent*:
1. *T is $\aleph_0$-categorical*
2. *T does not satisfy the finite cover property* (f.c.p.)
3. *there is $k \in \omega$ such that, for all $a \in U$, $E(U,a)$ has either $\leq k$ or infinitely many elements.*

Since, for every theory T, T's being $\aleph_1$-categorical implies T's being nmd and ω-stable (where 'nmd' signifies nonmultidimensionality), and this implies T's being ω-stable and without the f.c.p., it follows that, in the case of a unique equivalence relation,

$$T \text{ is } \aleph_1\text{-categorical} \Rightarrow T \text{ is } \aleph_0\text{-categorical}.$$

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