

## Remarks on Special Lattices and Related Constructive Logics with Strong Negation

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**Abstract** The main purposes of this paper are to provide an algebraic analysis of a certain class of constructive logics with strong negation, and to investigate algebraically the relations between strong and nonconstructible negations definable in these logics. The tools used in this analysis are varieties of algebraic structures of ordered pairs called *special N-lattices* which were first introduced as algebraic models for Nelson's *constructive logic with strong negation* (CLSN). Via suitable restrictions of the domains, algebras of this type are shown to be algebraic models for the propositional fragments of  $E^0$ , CLSN, Intuitionistic Logic, and  $E_+^0$ . The differences between  $E^0$  and CLSN are then studied, via the interrelations that the related algebras exhibit among strong and nonconstructible negations, properties of filters, and their behavior with respect to classically valid formulas.

**Introduction** Constructive Logic with Strong Negation (CLSN) was introduced to correct certain nonconstructive properties of Intuitionistic negation (see Nelson [5], Thomason [11], and Vorob'ev [13]). The logics  $E^0$ ,  $E_+^0$ , and  $E^+$  (henceforth *Effective Logics* (EL)) were introduced in Miglioli et al. [4] in order to study the concept of effectiveness in Computer Science. Though thought to be independent, CLSN and EL are in fact very close. As a matter of fact, the propositional fragment of  $E^0$  has the same axioms as CLSN (see Rasiowa [9], Chapter XII) without the axioms for weak negation and strong implication, plus the following rules:

$$\begin{array}{ccc}
 \begin{array}{c} [A] \quad [A] \\ \vdots \quad \vdots \\ B \quad \neg B \end{array} & \begin{array}{c} \hline \neg T A \end{array} & \begin{array}{c} [\neg A] \quad [\neg A] \\ \vdots \quad \vdots \\ \neg B \quad \neg B \end{array} \\
 (\neg T \text{ Int}) & & (T \text{ Int}) \quad \begin{array}{c} \hline T A \end{array}
 \end{array}$$

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\*I would like to thank Professor D. Vakarelov and Professor P. A. Miglioli for their criticisms and suggestions.