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An Equivalent of the Axiom of Choice in Finite Models of the Powerset Axiom

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Abstract It is shown that in a finite model for the set-theoretical Powerset axiom every set s has a Choice set iff every set s has a Meet set $\cap s$. Moreover, the Choice set of s is unique and is equal to $\cap s$, where $\cap s$ is a singleton and $\cap s \in s$.

Let $(F, \in)$ be a finite model for the set-theoretical Powerset axiom, i.e., in $(F, \in)$ every set has a powerset.

For instance, let us consider the finite model $(M, \in)$ whose domain consists of the three sets a, b, c and where the $\in$-relation is defined by:

(1) $a = \{b\}, \qquad b = \{a\}, \qquad c = \{a, b, c\}.$

It can be readily verified that $(M, \in)$ is a model for the Powerset axiom. Indeed, we have:

(2) $\mathcal{P}(a) = b, \qquad \mathcal{P}(b) = a, \qquad \mathcal{P}(c) = c$

where $\mathcal{P}(x)$ stands for the Powerset of x, i.e., the set of all subsets (needless to say, which exist in $(M, \in)$) of x.

We verify (2), say, for c. From (1) it follows that each one of the three sets a, b, c is a subset of c. Moreover, since a, b, c are collected by c, it follows that c is the set of all the subsets of c in $(M, \in)$. Hence $\mathcal{P}(c) = c$ in $(M, \in)$.

In [1] it is shown that in a finite model for the Powerset axiom the set-theoretical Extensionality axiom also holds. Thus, the notions of "uniqueness" and "equality" used in the above, and the notations introduced in (1) and (2), are justified.

Also, in [1] it is shown that in a finite model $(F, \in)$ of the Powerset axiom, for every set x and y

(3) $x \subseteq y$ iff $\mathcal{P}(x) \subseteq \mathcal{P}(y)$, and
(4) Every set of $(F, \in)$ is a powerset of some set of $(F, \in)$ and thus there is no empty set in $(F, \in)$.

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