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Negative Membership

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Abstract Generalized sets whose characteristic functions may assume any integer value, positive or negative, are formalized in a first-order two-sorted theory MSTZ which contains an exact copy of ZFC and is relatively consistent.

Introduction By *negative membership* we mean the fact of belonging to a collection of objects a negative number of times. This concept extends the notion of belonging to a collection of objects any positive number of times, which has been formalized using multisets (see Blizard [1] and [2]). A *multiset*, or *mset*, is a collection of objects, called *elements*, in which elements are allowed to repeat. The number of times an element repeats in a multiset is called its *multiplicity*. The *cardinality* of a multiset is the sum of the multiplicities of its elements. In the multiset $[c,a,b,a,a,b]$ the element a has multiplicity 3, b has multiplicity 2, and c has multiplicity 1. We denote this multiset by $[a,b,c]_{3,2,1}$. The cardinality of $[a,b,c]_{3,2,1}$ is 6. A *set* is a multiset in which each element has multiplicity 1. We denote the set $[c,a,b]_{1,1,1}$ by $\{a,b,c\}$. It is assumed that elements of multisets have finite multiplicities but that the number of distinct elements in a multiset need not be finite. The concept of negative membership, or *negative multiplicity*, has been used and investigated in the literature (see [4], [6], [10], [16], [17], and [18]). We develop a first-order two-sorted theory MSTZ for multisets in which elements may have integer multiplicities (positive, negative, or zero). In MSTZ, $[a,b,c]_{-1,2,-4}$ denotes the unique multiset containing -1 copies of a, two copies of b, and -4 copies of c. The theory MSTZ is a generalization of the theory MST (see [1] and [2]). We show that MSTZ contains an exact copy of ZFC and that MSTZ is relatively consistent.

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