330
Notre Dame Journal of Formal Logic
Volume 31, Number 2, Spring 1990

On Generalizations of a Theorem of Vaught

BALÁZS BIRÓ*

Abstract This paper deals with the cylindric algebraic version of Vaught's theorem on the existence of prime models of atomic theories. It is proved that the algebraic version proved by Serény, which states that under certain conditions every isomorphism between two cylindric set algebras (*Cs*'s) is a lower base-isomorphism, extends to generalized cylindric algebras (*Gs*'s) although it does not extend to generalized weak cylindric set algebras (*Gws*'s); indeed, it is not true for weak cylindric set algebras (*Ws*'s).

In this paper we investigate the following question: To which subclasses of cylindric algebras can the algebraic version of Vaught's theorem (Theorem 2.3.4 of Chang and Keisler [6] or Proposition 3 below) concerning the existence of prime models of atomic theories be extended? A version of Vaught's theorem has already been stated for cylindric set algebras in Serény [11], according to which every isomorphism between two cylindric set algebras (*Cs*'s) satisfying certain conditions is a lower base-isomorphism. We prove that this theorem is true not only for *Cs*'s but for generalized cylindric set algebras (*Gs*'s) as well, although it is not true for generalized weak cylindric set algebras (*Gws*'s). Indeed, it is false for weak cylindric set algebras (*Ws*'s).

It is worth adding that we have already proved for *Cs*'s that Serény's theorem requires all the conditions given in its statement (see Biró [2], [3], and [4] and Biró and Shelah [5]).

Our treatment is based on the books *Cylindric Algebras*, Parts I and II, by Henkin, Monk, and Tarski ([7] and [8]), and *Cylindric Set Algebras* by Henkin, Monk, Tarski, Andréka, and Németi [9]. Here we recall only the notions connected with our central concepts. The background of the following definitions is in [8]. Throughout, α is an ordinal. Let U be any set and p any element of ${}^{\alpha}U$. A *cylindric set algebra* of dimension α (Cs_α) with base U or a *weak cylindric set algebra* of dimension α (Ws_α) with base U determined by p is a Boolean set algebra whose elements have α-sequences as points and whose unit element is the

*Research supported by Hungarian National Foundation of Scientific Research, grant no. 1810.

Received August 4, 1987; revised November 9, 1989