The Homogeneous Form of
Logic Programs with Equality

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Abstract Let P be a Horn clause logic program. We suppose that P is *symmetric* in the sense that if C is a clause in P whose head is $s = t$, then there is a clause C^* in P which is like C except for having the head $t = s$. The *homogeneous form* of a clause $p(t_1, \ldots, t_n) \leftarrow B_1, \ldots, B_q$ is $p(x_1, \ldots, x_n) \leftarrow x_1 = t_1, \ldots, x_n = t_n, B_1, \ldots, B_q$. The *homogeneous form* P' of P is the set of homogeneous forms of clauses of P. Let T be a set of axioms asserting the reflexivity, symmetry, transitivity, and congruence (with respect to the predicates of P) of $=$. Then $P \cup T$ is *goal equivalent* to $P' \cup \{x = x\}$; i.e., for any goal G, $P \cup T \cup \{G\}$ is unsatisfiable iff $P' \cup \{x = x\} \cup \{G\}$ is unsatisfiable. The main interest of the paper lies in its construction of the Herbrand model M and in the proof that M is the *minimal* Herbrand model of both $P \cup T$ and $P' \cup \{x = x\}$.

1 Introduction   In their analysis of Prolog II van Emden and Lloyd [1] introduce the notion of the homogeneous form of a Horn clause logic program.

Definition   The *homogeneous form* of a clause $p(t_1, \ldots, t_n) \leftarrow B_1, \ldots, B_q$ is

$$p(x_1, \ldots, x_n) \leftarrow x_1 = t_1, \ldots, x_n = t_n, B_1, \ldots, B_q$$

where $x_1, \ldots, x_n$ are distinct variables not appearing in the original clause.

Definition   Let P be a program. The *homogeneous form* P' of P is the set of homogeneous forms of the clauses in P.