

## The Homogeneous Form of Logic Programs with Equality

WILLIAM DEMOPOULOS\*

**Abstract** Let  $P$  be a Horn clause logic program. We suppose that  $P$  is *symmetric* in the sense that if  $C$  is a clause in  $P$  whose head is  $s = t$ , then there is a clause  $C^*$  in  $P$  which is like  $C$  except for having the head  $t = s$ . The *homogeneous form* of a clause  $p(t_1, \dots, t_n) \leftarrow B_1, \dots, B_q$  is  $p(x_1, \dots, x_n) \leftarrow x_1 = t_1, \dots, x_n = t_n, B_1, \dots, B_q$ . The *homogeneous form*  $P'$  of  $P$  is the set of homogeneous forms of clauses of  $P$ . Let  $T$  be a set of axioms asserting the reflexivity, symmetry, transitivity, and congruence (with respect to the predicates of  $P$ ) of  $=$ . Then  $P \cup T$  is *goal equivalent* to  $P' \cup \{x = x\}$ ; i.e., for any goal  $G$ ,  $P \cup T \cup \{G\}$  is unsatisfiable iff  $P' \cup \{x = x\} \cup \{G\}$  is unsatisfiable. The main interest of the paper lies in its construction of the Herbrand model  $M$  and in the proof that  $M$  is the *minimal* Herbrand model of both  $P \cup T$  and  $P' \cup \{x = x\}$ .

**1 Introduction** In their analysis of Prolog II van Emden and Lloyd [1] introduce the notion of the homogeneous form of a Horn clause logic program.

**Definition** The *homogeneous form* of a clause  $p(t_1, \dots, t_n) \leftarrow B_1, \dots, B_q$  is

$$p(x_1, \dots, x_n) \leftarrow x_1 = t_1, \dots, x_n = t_n, B_1, \dots, B_q$$

where  $x_1, \dots, x_n$  are distinct variables not appearing in the original clause.

**Definition** Let  $P$  be a program. The *homogeneous form*  $P'$  of  $P$  is the set of homogeneous forms of the clauses in  $P$ .

---

\*I wish to thank E. W. Elcock, Edward P. Stabler, and Peter Hoddinott for introducing me to the theory of logic programming. I wish to thank A. Abdallah for suggesting Section 4. I am especially indebted to Kwok Hung Chan for carefully reading earlier drafts and providing critical comments and helpful suggestions too numerous to detail. Research support by the Social Sciences and Humanities Research Council and by the IBM Corporation is gratefully acknowledged.