Notre Dame Journal of Formal Logic
Volume 31, Number 2, Spring 1990

Models for Inconsistent and Incomplete Differential Calculus

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Abstract In Section 2, a nilpotent ring is defined. In Section 3, nonclassical model theory is sketched and an incomplete model is defined. In Section 4, it is shown that the elements of equational differential calculus hold in this model, and a comparison with synthetic differential geometry is made. In Section 5, an inconsistent theory is defined with many, though not all, of the same properties.

1 Introduction This paper extends the nonclassical model theory for inconsistent first-order equational theories developed in [4], [6], and [7], to the case of inconsistent equational theories strong enough for a reasonable notion of differentiation. The aim is to show that inconsistency does not cripple such an equational differential calculus. There have been a number of calls recently for inconsistent calculus, some appealing to the history of the calculus in which inconsistent claims abound (see, e.g., [9]). However, inconsistent calculus has resisted development, for at least two reasons. First, the functional structure of fields interacts with inconsistency to produce triviality in even the purely equational part of first-order theories with terms of finite length (as pointed out in [6], [7], and [9]), in a way which standard contradiction-containment devices, such as weakening *ex contradictione quodlibet*, do not prevent. Stronger theories, those including set membership, terms of infinite length, order, limits, and integration are then infected with the same triviality. Second, the functional structure of inconsistent set theory remains difficult to control, and seems to require sacrifice of logical principles in addition to, and more natural than, *ex contradictione quodlibet* (see, e.g., [2], [5], or [8], pp. 178–180). But unless there

Received August 14, 1987; revised May 26, 1988