

## Gentzenizing Schroeder-Heister's Natural Extension of Natural Deduction

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**Abstract** Our purpose here is to provide an example of how the use of the Gentzen-type sequential calculus considerably simplifies a complex Natural Deduction formalism. The formalism is that of Schroeder-Heister's system of higher-order rules. We show that the notions of Schroeder-Heister's that are the most difficult to handle (discharge functions and subrules) become redundant in the Gentzen-type version. The complex normalization proof given by Schroeder-Heister can be replaced therefore by a standard cut-elimination proof. It turns out also that the unusual form of some of the elimination rules of Schroeder-Heister corresponds to the natural, standard form of antecedent rules in sequential calculi.

In this paper we provide a Gentzen-type formulation of Schroeder-Heister's system found in [7]. This system is important from both the philosophical and the practical points of view: Its philosophical importance is due to the characterization it provides for the intuitionistic connectives, while the practical one is due to the fact that its notion of higher-order rules and its method of treating the elimination rules were incorporated into the Edinburgh LF (a general logical framework for implementing logical formalisms on a computer, which was developed in the computer science department of the University of Edinburgh; see [3], [4]). We shall show that the notions of Schroeder-Heister's that are the most difficult to handle (discharge functions and subrules) become redundant in the Gentzen-type version. Schroeder-Heister's complex normalization proof in [6] can therefore be replaced by a standard cut-elimination proof. Moreover, the unusual form of some of Schroeder-Heister's elimination rules corresponds to the natural, standard form of antecedent rules in sequential calculi. We also believe that the sequential presentation sheds new light on the connection between Schroeder-Heister's higher-order rules and intuitionistic implication and on Schroeder-Heister's characterization of the intuitionistic connectives.

*Received June 6, 1986; revised September 15, 1988*