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The Fraenkel–Mostowski Method, Revisited

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Abstract Permutation models generated by isomorphic topological groups satisfy the same choice principles (Boolean combinations of injectively bounded statements). As an application the group J_p of p-adic integers is characterized: A monothetic linear group G generates a model that satisfies the same choice principles that hold in the model corresponding to J_p iff the G-model satisfies the well-orderable selection principle, and AC_q holds, q prime, iff $q \neq p$. The main result is a strengthening of a previous theorem of Pincus: All Fraenkel-Mostowski-Specker independence proofs concerning choice principles can be proved in finite support models.

1 Introduction In this note we comment on some aspects of the structure theory of permutation models which are related to their historical development fifty years ago. Following some ideas of Fraenkel, permutation models were invented by Lindenbaum and Mostowski [16], [17] as a device for proving independence results on the axiom of choice in ZFA set theory (a weakening of ZF which permits a set A of atoms). In [17] Mostowski used models with only finite supports. Later he extended this method to models with infinite supports [18], and finally Specker [24] presented the most general construction of a permutation model from a group-generated Hausdorff topological group. This line of research seems to have been based on the conviction that more general construc-

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