44
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A Simplification of the Completeness Proofs for Guaspari and Solovay's R

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Abstract Alternative proofs for Guaspari and Solovay's completeness theorems for R are presented. R is an extension of the provability logic L and was developed in order to study the formal properties of the provability predicate of PA occurring in sentences that may contain connectives for witness comparison. (The primary example of sentences involving witness comparison is the Rosser sentence.) In this article the proof of the Kripke model completeness theorems employs tail models, as introduced by Visser, instead of the more usual *finite* Kripke models. The use of tail models makes it possible to derive arithmetical completeness from Kripke model completeness by *literally* embedding Kripke models into PA. Our arithmetical completeness theorem differs slightly from the one proved by Guaspari and Solovay, and it also forms a solution to the problem (advanced by Smoryński) of obtaining a completeness result with respect to a variety of orderings.

Introduction This paper deals with the completeness proofs given in [2] for the theory R, which is an extension of the provability logic L introduced in [5]. R is formulated in a language containing $\leq$ and $<$ as connectives for witness comparison, which enables one to study the provability predicate as it occurs in formulas like $\mathrm{Pr}(\ulcorner A \urcorner) < \mathrm{Pr}(\ulcorner B \urcorner)$, defined as $\exists x(\mathrm{Proof}(x, \ulcorner A \urcorner) \wedge \forall y \leq x \neg\mathrm{Proof}(y, \ulcorner B \urcorner))$. Rosser sentences are good examples of this kind of formula.

In Section 1 we describe the theories L and R and the alternative complete-

*The treatment of the Kripke model completeness theorem given in Section 2 has benefited much from [3]. Albert Visser's suggestions, corrections and remarks also made a substantial contribution to this paper.

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