Russell, Logicism, and the Choice
of Logical Constants

MICHAEL BYRD

Abstract It is here argued that Russell's *Principles of Mathematics* contains an intriguing idea about how to demarcate logical concepts from nonlogical ones. On this view, implication and generality emerge as the two fundamental logical concepts. Russell's 1903 proposals for defining other logical concepts from these basic ones are examined and extended. Despite its attractiveness, the proposal is ultimately unsatisfactory because of problems about defining negation and existential quantification.

Introduction   Traditional logicism holds that all mathematical concepts can be defined in terms of logical concepts and that all theorems of mathematics can be derived by logic from logical truths. Clearly, to assess the truth and philosophical significance of logicism, we must know what concepts are logical concepts and what truths are logical truths. Bertrand Russell held, for example, that the Axiom of Infinity, while perhaps true, was not a logical truth ([15], p. viii). Of equal importance is the question of what concepts are logical concepts. On this matter noted authorities such as Quine, Tarski, and Church have made pessimistic assessments. Quine standardly characterizes the logical truths as truths which involve only the logical words essentially. He enumerates the logical words (e.g., "not", "or", "all", but naturally not "necessarily"). But he refuses to go beyond enumeration: "Logical vocabulary is specified only I suppose by enumeration" ([12], p. 141). Tarski regarded it as quite possible that future investigations would compel us to hold that the division of terms into logical and extralogical was, to a greater or lesser degree, arbitrary ([19], p. 420; see also Wang [20], p. 54, and Church [2], p. 58, note 129). Naturally, such views challenge the philosophical importance of even the weaker logicist thesis that all mathematical concepts are definable in purely logical terms.

I shall, in what follows, examine a response to the problem of demarcating the logical constants which is implicit in the first English presentation of logi-