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The Boolean Spectrum of an o-Minimal Theory

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Abstract We show that the number of isomorphism types of Boolean algebras of definable subsets of countable models of an *o*-minimal theory is either 1 or $2^{\aleph_0}$. We also show that the number of such isomorphism types is 1 if and only if no countable model of the *o*-minimal theory contains an infinite discretely ordered interval.

A structure $\mathfrak{M}$ linearly ordered by $<$ is said to be *o-minimal* if its definable subsets are exactly those that can be obtained by using only quantifier-free formulas involving $<$, i.e., unions of finitely many points and intervals. A complete theory **T** of linearly ordered structures is said to be *o*-minimal if all models of **T** are *o*-minimal. We note that in [2] and [5] it is shown that "all models" may be replaced by "some model" in the definition of an *o*-minimal theory. Model theoretically, *o*-minimal structures are the simplest linearly ordered structures, playing the same role with respect to $<$ as minimal structures do with respect to $=$. Carrying this analogy further, *o*-minimal theories correspond to strongly minimal theories.

o-minimal theories were studied extensively in [4]. Here we wish to consider a particular question about such theories. Let **T** be a theory and $\mathfrak{M}$ a model of **T**. Denote by $B(\mathfrak{M})$ the Boolean algebra of the definable subsets of $\mathfrak{M}$, and define the Boolean spectrum of **T**, Spec**T**, to be the set of isomorphism types of the algebras $B(\mathfrak{M})$ as $\mathfrak{M}$ ranges over the countable models of **T**. It is well known that the Boolean spectrum of a strongly minimal theory **T** contains only one element: the isomorphism type of the countable superatomic algebra of CB-type (2,1). That is, a strongly minimal theory **T** is p-$\aleph_0$-categorical (see [7]). Thus we are interested in examining the corresponding problem in the *o*-minimal case.

The most obvious question to raise is whether all *o*-minimal theories are

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