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Some Modal Logics Based on a Three-Valued Logic

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1 Introduction A K-modal logic based on Łukasiewicz's three-valued logic has been formulated by Schotch [2]. In this paper we formulate K-, M-, $S4$-, and $S5$-modal logics based on a general three-valued logic by using the notion of a matrix in [3].

In Section 2, we define truth values, formulas, and matrices. In Section 3, we introduce three-valued Kripke models defined in [1]. In Section 4, we present the systems K, M, $S4$, and $S5$ of modal logic based on a general three-valued logic (3-K_3, 3-K_2, 3-M_3, 3-M_2, 3-$S4_3$, 3-$S4_2$, 3-$S5_3$, and 3-$S5_2$). 3-K_3, 3-M_3, 3-$S4_3$, and 3-$S5_3$ are modal logics based on a three-valued logic in which the modal operators take on all three of our truth-values. 3-K_2, 3-M_2, 3-$S4_2$, and 3-$S5_2$ are modal logics based on a three-valued logic in which the modal operators take only the two classical truth-values. In Section 5, we develop the syntax of 3-K_i, 3-M_i, 3-$S4_i$, and 3-$S5_i$ ($i = 2,3$) and it will be shown that the cut-elimination theorems no longer hold in 3-K_i, 3-M_i, 3-$S4_i$, and 3-$S5_i$. In Section 6, we prove the completeness theorems for 3-K_i, 3-M_i, 3-$S4_i$, and 3-$S5_i$.

2 Matrices

2.1 Truth values We take 1, 2, and 3 as truth-values. Intuitively '1' stands for 'true' and '3' for 'false', whereas '2' may be interpreted as 'undefined' or 'meaningless'.

We denote the set of all the truth values by **T**. $\mathbf{T} = \{1,2,3\}$.

2.2 Primitive symbols

(1) Propositional variables: p, q, r, etc.
(2) Propositional connectives:

$$F_i(*_1, \ldots, *_{\alpha_i}) = i = 1,2, \ldots, \beta, \alpha_i \geqq 1.$$

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