

## Book Review

Forster, T. E., *Set Theory with a Universal Set: Exploring an Untyped Universe*, Oxford Logic Guides Vol. 20, Oxford University Press, New York, 1992. 152 pages.

*I* As the title suggests, this book is an 'exploration' of the set theoretic universe (or universes) under the assumption that there is a set  $V$  of all sets. Although several different approaches to  $V$  are touched upon, most of the book concerns Quine's theory NF. Indeed the theory NF is, in the reviewer's opinion, by far the most interesting part of the story. The book's coverage of NF in Chapter 2, its exceptional bibliography, and especially its coverage of the Rieger-Bernays permutation model construction described in Chapter 3, makes this book the most complete exposition of NF and its models yet published.

Many readers may be put off by the unconventional notation, and may even be slightly alarmed by the rather casual (and occasionally imprecise) terminology used. The author clearly feels, however, that in a subject as broad as a study of the disparate theories that admit the set  $V$  it is the ideas that should take key position, and these will be found in abundance. Thus, to give a familiar example, Cantor's theorem that there is no surjection of a set  $X$  onto its powerset  $P(X)$  is clearly going to fail for  $X = V$  because  $P(V) \subseteq V$  but the idea behind the proof can be readily transformed to give the result in NF that there is no surjection from the set of singletons of  $X$  into  $P(X)$  and its corollary that the cardinality of the set of all singletons is strictly smaller than the cardinality of  $V$  itself. (Perhaps this example shows why the often seemingly paranoiac obsession with the 'paradoxes' is of value: often the arguments do seem to be telling us something important about the theories and those sets that they describe.) This book contains a vast number of interesting arguments of varying degrees of difficulty—not only unfamiliar forms of familiar ones—which any reader interested in the subject would do well to assimilate.

This said, the book is not always easy to read: the sequence of definitions and results chosen does not always seem to be ordered in the most natural way, and familiar definitions and concepts are mixed with the unfamiliar ones often without indication of which is which or without preparation for the reader to whom this subject may be quite novel. Within its field, the scope of the book is almost encyclopedic, but at times the reviewer (when trying desperately to find

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