Notre Dame Journal of Formal Logic
Volume 34, Number 2, Spring 1993

BCK and BCI Logics, Condensed Detachment and the 2-Property

J. ROGER HINDLEY

Abstract Some of the main properties of the BCK and BCI logics of implication are summarized, focusing on their connections with their condensed logics and with combinators and lambda-calculus. (A condensed logic is the set of all formulas deducible from the logic's axioms by the condensed detachment rule of Carew Meredith.) A full proof is given of the preservation of the 2- and 1-2-properties by condensed detachment, based on ideas of S. Jaskowski.

1 Introduction and notation

1.0 Introduction This article is a summary of some of the main properties of implicational BCK and BCI logics, focusing on their connections with combinators and their condensed logics. (A condensed logic is the set of all formulas provable from the logic's axioms by Carew Meredith's rule of condensed detachment.)

The material outlined here will not be new; it will mainly be from the work of N. D. Belnap, M. W. Bunder, S. Hirokawa, and R. K. Meyer, but above all from that of S. Jaskowski.

The key definitions will be sketched in Section 1, then BCK-logic will be treated in Section 2 and BCI-logic in Section 3.

A proof of the preservation of the 2- and 1-2-properties by condensed detachment will be given in Section 4. It will be based on ideas from Jaskowski [24]. (A formula has the 2-property when each variable in it occurs exactly twice, and the 1-2 property when each variable in it occurs at most twice.) The first published proof of the preservation of the 2-property is in Belnap [3], which is probably not widely available now.

Most proofs other than the 2- and 1-2-preservation proof will be omitted. Detailed references to the literature will be included, however.

Received December 10, 1990; revised January 15, 1992