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An Arithmetical Completeness Theorem for Pre-permutations

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Abstract We prove an extension of an arithmetical completeness theorem for the system $\mathbf{R}^\omega$ with respect to pre-permutational arithmetic interpretations to all modal sentences. Hitherto, this type of completeness theorem has only been given for modal sentences with no nestings of witness comparisons.

In their joint paper [1], Guaspari and Solovay provide a modal analysis of Rosser sentences. Their results are presented, discussed, and somewhat complemented in great detail in Chapter 6 of Smoryński's recently published book [2]. Both for the sake of shortness and convenience, we assume full familiarity with this exposition and will refer to it throughout this paper.

Among other results, Guaspari and Solovay prove an Arithmetical Completeness Theorem (ACT) for the modal system $\mathbf{R}^\omega$ which is briefly described as follows ([2], p. 259–262).

The *language* of $\mathbf{R}^\omega$ is the usual one for propositional logic but equipped with witness comparisons $\leq$ and $<$.

The *axioms* of $\mathbf{R}^\omega$ are all sentences (A1–A7) together with the necessitations of (A1–A6).

(A1) All tautologies
(A2) $\Box A \wedge \Box(A \to B) \to \Box B$
(A3) $\Box A \to \Box\Box A$
(A4) $\Box(\Box A \to A) \to \Box A$
(A5) $A \to \Box A$, for all Σ-formulas ([2], p. 260)
(A6) Order axioms for $\leq$ and $<$ ([2], p. 261)
(A7) $\Box A \to A$.

(The necessitation of a sentence A is $\Box A$.)

The only *rule of inference* is modus ponens.

An *arithmetic interpretation* is an assignment $*$ of arithmetic sentences p^* to modal atoms p that extends as follows:

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