A Functional Partial Semantics
for Intensional Logic

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Abstract In this paper a partial semantics for the higher order modal language of Intensional Logic is suggested. Partial semantic values of functional types are defined as monotone functions on partially ordered sets; it is shown that this characterization is materially adequate for representing partial values and that it overcomes the difficulties that arise when we attempt to introduce one-place partial functions in the hierarchy of types. Partial values of any type are related to classical values of the same type by means of a relation of approximation. This allows us to compare partial models with classical models. Classical semantics then appears to be a part of partial semantics to the extent that there exists a bijective mapping from classical models onto totally defined partial models. This also allows us to define, according to the partial semantics, a notion of entailment which is coextensive with the classical notion.

1 Introduction   Even though much work has been done in partial semantics for propositional and quantified first order languages, little has been said about partial semantics for higher order languages. The first attempt to introduce partiality in Type Theory of which we are aware goes back to Tichý [11]. There is also the interesting work of Muskens [9], which suggests a partialized version of Montague's semantics. Musken's semantics is relational, in the sense that partial semantical objects are defined as partial relations, not as partial functions. This strategy is in large part justified by the apparent impossibility, discussed in this paper, of coding partial relations or partial many-place functions by one-place functions, as Schönfinkel's theorem might suggest. Note that the same problem has motivated Tichý's own strategy, which consists of considering only many-place partial functions. For our point, we do not claim that Tichý's and Musken's strategies are inadequate, nor do we think they are uninteresting. Our claim is merely that it is possible, using only one-place functions, to construct a partial semantics for the higher-order modal language of Intensional Logic.