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A Sequent- or Tableau-style System for Lewis's Counterfactual Logic VC

IAN PHILIP GENT

Abstract In a 1983 paper, de Swart gave sequent based proof systems for two counterfactual logics: Stalnaker's **VCS** and Lewis's **VC**. In this paper I demonstrate that de Swart's system for **VC** is incorrect by giving a counterexample. This counterexample does not affect de Swart's system for **VCS**. Then I give a new sequent- or tableau-style proof system for **VC** together with soundness and completeness proofs. The system I give is closely modeled on de Swart's.

1 Introduction Lewis [2] presented a counterfactual logic **VC**. de Swart [1] presented first a sequent-based proof system for Stalnaker's counterfactual logic **VCS**, together with soundness and completeness proofs, and then a proof system for **VC**. Unfortunately, the soundness and completeness proofs for **VC** were only sketched. In this paper I show that de Swart's system for **VC** is incorrect, in that there is a **VC**-valid formula which the system reports to be invalid. This paper concentrates exclusively on **VC**; de Swart's work on **VCS** is not affected by the counterexample to **VC**.

In the rest of this section, I very briefly introduce Lewis's logic **VC**. In Section 2 I describe de Swart's system for **VC** and in Section 3 I give a counterexample to this system. In Section 4 I give a new proof system for **VC**, for which I give soundness and completeness proofs in Sections 5 and 6 respectively. This system cannot exactly be described as either a tableau or a Gentzen sequent system but is closely related to both. It differs from tableau systems in that nodes in a derivation tree are labeled with sets of formulas rather than single formulas, but it differs from Gentzen sequent systems in that I do not use the sequent arrow, preferring the semantic notion of signed formulas.

The language of **VC** contains standard propositional connectives $\wedge$, $\vee$, $\neg$, $\supset$, the propositional constants $\top$ and $\bot$, and the extra connectives $\leq$ and $\Box\!\!\rightarrow$. $\mathcal{A} \leq \mathcal{B}$ is read as "$\mathcal{A}$ is at least as possible as $\mathcal{B}$". The connective $\Box\!\!\rightarrow$ is used for

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