Notre Dame Journal of Formal Logic
Volume 34, Number 4, Fall 1993

BREAKUP: A Preprocessing Algorithm for Satisfiability Testing of CNF Formulas

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Abstract An algorithm called BREAKUP, which processes CNF formulas by separating them into "connected components," is introduced. BREAKUP is then used to speed up the testing of some first-order formulas for satisfiability using Iwama's **IS** Algorithm. The complexity of this algorithm is shown to be on the order of $O(nc \cdot nv)$, where nc is the number of clauses and nv is the number of variables.

1 Introduction Connectedness is an attribute of graphs that can be useful in deciding the satisfiability of propositional formulas in conjunctive normal form (CNF). Preprocessing to separate a formula into "connected components" is generally fast and easy, and can significantly speed up testing of the formula for satisfiability. BREAKUP, the algorithm introduced here, will be used in conjunction with Iwama's **IS** Algorithm [4],[5] and will be shown to significantly speed up (as compared to using **IS** alone) the processing of some formulas from Gilmore [3] and Davis and Putnam [2].

A CNF formula is a conjunction of clauses where each clause is a disjunction of literals. A literal is a propositional variable or the negation of a propositional variable. Given a pair of literals $\{p, \neg p\}$, each will be called the *mate* of the other (cf. Davis [1]). Clearly, we can assume that no clause contains both a literal and its mate. There are two ways to think of clauses in a formula as connected. First, two clauses c, d, are *connected* if there exists a chain of clauses starting with c and ending with d such that for any two adjacent clauses x, y, either y contains a literal that occurs in x or y contains the mate of a literal that occurs in x. Alternatively, c and d can be said to be *connected* if there is a chain of clauses starting with c and ending with d such that if x and y are adjacent then y contains the mate of a literal that occurs in x. In either case, a set of clauses is said to be *connected* if every pair in the set is connected and a *component* is a maximally connected set of the clauses of the formula.

Received July 15, 1992; revised March 15, 1993