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# The Gupta-Belnap Systems $\mathbf{S}^{\#}$ and $\mathbf{S}^{*}$ are not Axiomatisable

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**Abstract** Anil Gupta and Nuel Belnap's *The Revision Theory of Truth* presents revision theoretic systems of circular definitions. Part I of the present paper shows that the revision theories $\mathbf{S}^{\#}$ and $\mathbf{S}^{*}$ are not axiomatisable. Part II refines this result. Among other things, Part II shows that there is a strong relationship between revision theories and the theory of inductive definitions. This relationship is exploited to show that $\mathbf{S}^{\#}$ and $\mathbf{S}^{*}$ (and all "plausible" revision theories of circular definitions) are of complexity at least $\Pi^1_2$.

***1 Introduction The Revision Theory of Truth*** Gupta and Belnap [2] treats "____ is true" as a predicate of sentences. (Strictly speaking, "is true" can be meaningfully applied to non-sentences. "Tracy is true" is well-formed and false. " 'Snow is white' is true" is well-formed and true.) Furthermore, [2] takes the corresponding concept, *truth*, to be a *circular* concept: in the definition of "is true", the expression "is true", which is the definiendum, appears in the definiens.

More precisely, truth is defined by the set of partial definitions of the form

'$p$' is true $=_{\mathrm{Df}} p$

where $p$ ranges over the sentences of the language. The definition of truth insofar as truth applies to "snow is white" is not circular. The pertinent definition is

'snow is white' is true $=_{\mathrm{Df}}$ snow is white,

and the definiendum ("is true") does not occur in the definiens. But the definition of truth insofar as truth applies to "what Tracy says is true" *is* circular. The pertinent definition is

'what Tracy says is true' is true $=_{\mathrm{Df}}$ what Tracy says is true,

and the definiendum ("is true") *does* occur in the definiens.

Before considering the special behaviour of truth, [2] develops general semantic theories of circularly defined concepts: revision theories $\mathbf{S}_0, \mathbf{S}_1, \ldots, \mathbf{S}_n, \ldots$ and $\mathbf{S}^{\#}$

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