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Zermelo, Reductionism, and the Philosophy of Mathematics

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Abstract Whereas Zermelo's foundational program is implicitly reductionist, the precise character of his reductionism is quite unclear. Although Zermelo follows Hilbert methodologically, his philosophical viewpoint in 1908 is broadly at odds with that of Hilbert. Zermelo's interest in the semantic paradoxes permits an intuitive concept of mathematical definability to play an important role in his formulation of axioms for set theory. By implication, definability figures in Zermelo's philosophical concept of set, which is seen to be nonstructural in character. Zermelo's advocacy of universal definability is intended to blunt tensions between platonists and constructivists. Finally, the method of justification of mathematical axioms is taken to be of an empirical and public character, at least in part, and, as a consequence, threatens Zermelo's foundational program.

What foundational role, if any, is set theory to play? One relatively straightforward answer has come to be called reductionism. Let us take reductionism to encompass the following claims:

(R1) All mathematical objects are sets.
(R2) All mathematical concepts are definable in terms of membership.
(R3) All mathematical truths are set-theoretic truths.

Reductionism embraces set theory as the metaphysical foundation of the mathematical sciences: mathematical objects, being sets, have whatever sort of reality sets have. We note that, at least according to one common understanding of what it is for a proposition to be true, (R3) is not independent of (R1) and (R2): it is not clear that it makes any sense to affirm (R3) while denying (R1) and (R2), or vice versa. Also, as it stands, (R2) remains vague in that the nature of the definability involved here is left entirely open.

At the other extreme is the view—now widely if not universally accepted—that set theory, despite its important subject, can be foundational only in that

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