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Book Review

Geoffrey Hellman. *Mathematics Without Numbers*. Oxford University Press, Oxford, 1989. ix+154 pages.

1 *Introduction* The present book attempts to arrange a marriage between two traditions in the philosophy of mathematics which have arguably always belonged together. One tradition, now usually termed ‘structuralism’, is about a century old and originates in Dedekind’s 1888 article “Was sind und was sollen die Zahlen” (reprinted in [5]). In this paper, Dedekind argues that the mathematical content of number theory is invariant under transformations defined on its subject matter which preserve arithmetical structure. More generally, and more vaguely, the structuralist view is that mathematics is *about structure*: that the mathematical content of an assertion or theory is invariant under isomorphisms of interpretations of that assertion or theory.

The other tradition, of more recent vintage, is sometimes called ‘modalism’. In general terms, this is the view that classical mathematics is (covertly) modal in character; that the language of classical mathematics makes assertions about what *would* hold in any structure of a certain sort, but does not assert the actual existence of any such structure. The view originates, as far as I can tell, with Putnam in [11]. Formulated with reference to set theory, the view is that a statement is equivalent to a modal assertion saying that its first-order representation holds in every possible standard model of the relevant rank (if the statement is of bounded rank; a more complex use of modal notions leads to an interpretation of statements of unbounded rank). The models in question are normally construed as possible *concrete* structures: in Putnam’s version, possible physical realizations of certain directed graphs.

The synthesis of these positions leads roughly to the following view. Putnam is clearly not especially interested in physical realizations of graphs in terms, say, of pencil points and arrows. Rather, the thesis is that these are of interest only because they exemplify a certain structure or isomorphism type. The proper formulation of modalism should rather make reference to *all possible realizations whatever* of the relevant isomorphism type. That is to say that the language of classical mathematics

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