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# Introduction to the Special Issue on Vaught's Conjecture

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Vaught's Conjecture, stated in Vaught's 1961 paper [3], "Denumerable models of complete theories," says that for a countable complete elementary first-order theory, the number of countable models, up to isomorphism, is either countable or $2^{\aleph_0}$. The conjecture has attracted a great deal of attention over the past 40 years, inspiring work in model theory and in other branches of logic as well.

In May 2005, the Logic Group at the University of Notre Dame held a workshop to discuss the ongoing forward progress of resolving Vaught's Conjecture. The idea was to bring together a number of people from different areas of mathematical logic and allow them to focus together on Vaught's Conjecture. The result was, by all accounts, an outstanding workshop.

This refereed special issue is one of the outcomes of this workshop. Not only does this issue contain the writeup of some of the talks given at the workshop but two of the papers (Marker [2], p. 93 and Hjorth [1], p. 49) solve questions which arose during the workshop.

Some of the talks (for example, by Marker and by Buechler) focused on settings in which Vaught's Conjecture holds. Some of the talks and resulting papers focused on the features that a counterexample would necessarily have, and on examples illustrating at least some of those features. The "Scott rank" of a countable structure is a measure of internal model theoretic complexity. Morley showed that if $T$ is a counterexample to Vaught's Conjecture, then $T$ has only countably many models of any given countable rank, and it has models of arbitrarily large countable rank. Thus, the number of countable models of $T$, up to isomorphism, is $\aleph_1$. Sacks's talk described a chain of models that a counterexample would have, of increasing Scott rank, preserving satisfaction for increasingly large admissible fragments.

The speakers at the workshop were, in the order in which they spoke, Gerald Sacks (2 presentations), Su Gao, David Marker (2 presentations), Wesley Calvert, Valentina Harizanov, Robin Knight, Łudomir Newelski (2 presentations), Nate

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