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ALTERNATIVE COMPLETENESS THEOREMS FOR MODAL SYSTEMS

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Since the development of semantics for the modal systems T, S4 and S5 notably in [1] there have appeared several completeness theorems for these systems. Some, e.g. [1][1] rely on the method of semantic tableaux which are shewn to give a decision procedure. A simpler proof, though one not yielding to decision procedure, follows from [3][2]. In part I of this paper we shew, by extensions of known results, some relations between the completeness of S5, S4 and T. In part II we shew how Anderson's [5] decision procedure for T can yield a relatively simple completeness proof for that system.

I. The system T (v.[6]) is a system of propositional modal logic based on the following additions to some standard axiomatic basis for the propositional calculus, (L for necessity);

LA1 $Lp \supset p$
LA2 $L(p \supset q) \supset (Lp \supset Lq)$
LR1 $\vdash \alpha \rightarrow \vdash L\alpha$

S4 is T with the addition of

LA3 $Lp \supset LLp$

S5 is T with the addition of

LA4 $\sim Lp \supset L \sim Lp$

We define validity in T, S4, and S5 in the manner of [1] as truth in all T, S4, S5 models. A T-model is an ordered triple $\langle VWR \rangle$ where W is a set of objects (worlds), R a reflexive relation over W, and V a function (assignment) taking as arguments a.) wffs of T b.) members of W and as values the truth values 1 or 0, and satisfying the following:

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