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DECISION FOR K4

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It was asked in [1] whether K4 contained K5. We show that it does, and give a decision procedure for the system, which has the third degree of completeness. To this end we establish a system SR which turns out to be an alternative version of K4. As a basis we take propositional calculus, **PC**, with substitution and C-detachment, and the axioms:

1. *RCpRp*
2. *CRNpNRp*
3. *CNRpRNp*
4. *CRCpqCRpRq*

with the rule to infer $R\alpha$ from α ($\mathcal{R}$).
Having **PC**, 2-4, we obviously have the meta-rule:
To infer $\phi\beta$ from $E\alpha\beta$ and $\phi\alpha$ (**EXT**).

5.	*ENRpRNp*	[2, 3
6.	*CRpRRp*	[4 *q/Rp*, 1
7.	*CRRpRp*	[6 *p/Np*, 5, **EXT**, **PC**
8.	*ERpRRp*	[6, 7
9.	*CNRCpqNCRpRq*	
	Dem. (*1*) *CNRCpqRNCpq*	[**PC**, 5
	(*2*) *CRNCpqRp*	[**PC**, $\mathcal{R}$, 4
	(*3*) *CRNCpqRNq*	[**PC**, $\mathcal{R}$, 4
	(*4*) *CRNCpqNRq*	[(3), 5
	(*5*) *CRNCpqNCRpRq*	[(2), (4)
	Prop.	[(1), (5)
10.	*ERCpqCRpRq*	[4, 9

With 5, 8, 10 and **EXT** we can reduce every expression to an inferentially equivalent set of forms

$$\text{(I)}\ C\alpha_1, \ldots\ldots C\alpha_n\beta$$

with each α_i an elementary variable or such negated, or either of those preceded by R, and β a variable not appearing as a component in any α_i.

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