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# ON THE EXTENSION OF S4 WITH *CLMpMLp*

R. A. BULL

Over the last few years various logicians have considered the modal system obtained by extending S4[1] with

$$CLMpMLp,$$

but no demonstration that the system is decidable, or description of a characteristic model for it, has been published.[2] The purpose of this paper is to fill this gap by showing the system to have the finite model property—so that it is decidable (by [2], Lemma 4) and characterized by order closure models (by [1], Lemma 1)—and obtaining a characteristic order closure model for it. I assume familiarity with closure algebras (see [3] and [4]), with the order closure models of [1], and with the finite model property (see [2]). I do not distinguish between a closure algebra and the model obtained from it by designating the unit element; a closure algebra can be regarded as a Boolean algebra with a closure operator defined on it, and this representation is the most convenient for my purposes. I use the symbol - for relative complement, instead of in its normal role of complement proper; and I use the interior operator **I** (complement of closure of complement).

(It may be of interest that the system can also be obtained by extending S4 with either of the rules

$$\vdash M\alpha \Longrightarrow \vdash ML\alpha$$
$$\vdash M\alpha,\ \vdash M\beta \Longrightarrow \vdash MK\alpha\beta.$$

To prove this I derive them in rotation:

(a) Given $CLMpMLp$,
$$\vdash M\alpha \Longrightarrow \vdash LM\alpha$$
$$\Longrightarrow \vdash LM\alpha,\ \vdash CLM\alpha ML\alpha$$
$$\Longrightarrow \vdash ML\alpha.$$

(b) Given $\vdash M\alpha \Longrightarrow \vdash ML\alpha$, since $\vdash_{S4} CLMLpCLMLqMKpq$,

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