Notre Dame Journal of Formal Logic
Volume VIII, Number 4, October 1967

REGRESSIVE FUNCTIONS AND COMBINATORIAL FUNCTIONS

CARL E. BREDLAU

1. *Introduction.** Let ε denote the set of all non-negative integers and let ε^* denote the set of all integers. Every function $f(n)$ from ε into ε uniquely determines a function c_i from ε into ε^* such that

$$f(n) = \sum_{i=1}^{n} c_i \binom{n}{i}, \quad \text{for } n \in \varepsilon. \tag{1}$$

The function $f(n)$ is called *combinatorial* if the function c_i related to $f(n)$ by (1) assumes no negative values. The function c_i is called the *associated function* of $f(n)$. The function c_i can be explicitly expressed in terms of the function $f(n)$ by the formula:

$$c_n = \sum_{i=1}^{n} (-1)^i \binom{n}{i} f(n-i). \tag{2}$$

Combinatorial functions were introduced by Myhill in a set-theoretic manner in [3] and play a fundamental role in the theory of recursive equivalence types; however, in what follows we need only the number-theoretic definition of a combinatorial function given above.

We note that if c_i is an effectively computable function (or formally, a recursive function), so is $f(n)$. For given n we can effectively calculate $c_0, \ldots, c_n$ and hence $f(n)$ by (1). Conversely, if $f(n)$ is a recursive combinatorial function, we can, given n, compute $f(0), \ldots, f(n)$, and hence c_n by (2). Thus c_i is a recursive function if $f(n)$ is. We conclude that for a combinatorial function $f(n)$,

$$f(n) \textit{ is recursive} \Longleftrightarrow c_i \textit{ is recursive.}$$

A function t_n from ε into ε is *regressive*, if it is one-to-one (1-1) and there exists a partial recursive function $p(x)$ such that

$$\rho t \subset \delta p, \tag{3}$$

$$(\forall n)[p(t_n) = t_{n \dot{-} 1}]. \tag{4}$$

*Research on this paper was done during 1964-65 under the direction of Dr. J. C. E. Dekker, while the author was a Henry Rutgers Scholar.

Received September 21, 1966