Notre Dame Journal of Formal Logic
Volume VIII, Number 4, October 1967

A SINGLE AXIOM FOR THE MEREOLOGICAL NOTION OF PROPER PART

CZESŁAW LEJEWSKI

The mereological notion of proper part was used by Leśniewski as a primitive, *i.e.*, undefined, notion in his first system of mereology constructed in 1915.[1] In terms of a language which is a slightly modified version of the Peano-Russellian symbolism, the axiomatic presuppositions of this system,—to be referred to as System 𝔄,—can be stated as follows:

AA1 $[AB]:A\varepsilon\mathbf{pt}(B).\supset.B\varepsilon\sim(A)$

AA2 $[ABC]:A\varepsilon\mathbf{pt}(B).B\varepsilon\mathbf{pt}(C).\supset.A\varepsilon\mathbf{pt}(C)$

AD1 $[AB]\therefore A\varepsilon A:A=B.\lor.A\varepsilon\mathbf{pt}(B)\therefore\equiv.A\varepsilon\mathbf{el}(B)$

AD2 $[Aa]::A\varepsilon A\therefore[B]:B\varepsilon a.\supset.B\varepsilon\mathbf{el}(A)\therefore[B]:B\varepsilon\mathbf{el}(A).\supset.[\exists CD].C\varepsilon a.D\varepsilon\mathbf{el}(B).$
$D\varepsilon\mathbf{el}(C)\therefore\equiv.A\varepsilon\mathbf{Kl}(a)$

AA3 $[ABa]:A\varepsilon\mathbf{Kl}(a).B\varepsilon\mathbf{Kl}(a).\supset.A\varepsilon B$

AA4 $[Aa]:A\varepsilon a.\supset.[\exists B].B\varepsilon\mathbf{Kl}(a)$

It is to be noted that *AA3* and *AA4* are stated with the aid of a defined notion, which means that they have to be preceded by an appropriate definition or by appropriate definitions. The practice of using defined notions for the purpose of stating axioms was soon abandoned by Leśniewski, who in 1918 constructed a new system of mereology,—I will refer to it as System 𝔅,—with the following axiomatic basis:

BA1 $[AB]:A\varepsilon\mathbf{pt}(B).\supset.B\varepsilon\sim(A)$

BA2 $[ABC]:A\varepsilon\mathbf{pt}(B).B\varepsilon\mathbf{pt}(C).\supset.A\varepsilon\mathbf{pt}(C)$

BA3 $[ABCa]:\therefore C\varepsilon a::[D]\therefore D\varepsilon a.\supset:D=A.\lor.D\varepsilon\mathbf{pt}(A)::[D]\therefore D\varepsilon a.\supset:D=B.\lor.D\varepsilon\mathbf{pt}(B)::$
$[D]\therefore D\varepsilon\mathbf{pt}(A).\lor.D\varepsilon\mathbf{pt}(B):\supset:[\exists E]:E=D.\lor.E\varepsilon\mathbf{pt}(D):E\varepsilon a.\lor.[\exists F].F\varepsilon a.$
$E\varepsilon\mathbf{pt}(F)::\supset.A\varepsilon B$

BA4 $[Aa]:\therefore A\varepsilon a.\supset::[\exists B]::[C]\therefore C\varepsilon a.\supset:C=B.\lor.C\varepsilon\mathbf{pt}(B)::[C]\therefore C\varepsilon\mathbf{pt}(B).\supset:[\exists D]:$
$D=C.\lor.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\lor.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)$

On subjoining to the above given axiomatic basis definitions *BD1*(=*AD1*) *BD2*(=*AD2*) it can be proved, as Leśniewski showed in his 'O podstawach matematyki'[2], that Systems 𝔄 and 𝔅 are inferentially equivalent to one another.

Received December 21, 1966