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Modal-Epistemic Variants of Shapiro's System of Epistemic Arithmetic

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Abstract This paper presents formalizations of classical first-order arithmetic which contain a modal and an epistemic operator. The embedding under variants of Gödel's translation of Intuitionistic arithmetic in such systems is discussed, and for one modal-epistemic system of arithmetic a possible worlds semantics is given.

1 Introduction This paper discusses modal-epistemic systems of arithmetic. They can be situated among the so-called "epistemic" formulations of arithmetic, of which Shapiro's system of epistemic arithmetic is a paradigmatic example. The next section reviews the basic properties of Shapiro's system. In the third section, it is explained how an analysis of the absolute provability operator of Shapiro's system naturally leads one to develop a modal-epistemic theory of arithmetic. Subsequently a modal-epistemic system of arithmetic (*MEA*) is presented. In the fourth section it is shown that the modal-epistemic system is a conservative extension of Heyting arithmetic. In the fifth section, a possible world semantics for the system is constructed. A soundness theorem is given and a completeness theorem is proved for the logical fragment of the system. In Section six, we exploit the surplus of expressive power of our modal-epistemic analysis of Shapiro's absolute provability operator in constructing a modal-epistemic system of arithmetic on a modal-structural basis. It is suggested that for the modal-structural interpretation of arithmetic to "make sense of" intuitionistic arithmetic, it must assume a stronger non-logical statement than it has hitherto been prepared to do.

2 Shapiro's system of Epistemic Arithmetic (EA) The language of *EA* (L_{EA}) contains all the symbols of the formal language of first-order arithmetic (L_{PA}), plus an epistemic sentence operator K (which we consider to be a logical symbol). So the only nonlogical symbols are the individual constant **0**, a one-place function symbol **s** (the successor operator), and the two-place function symbols + (plus) and · (times). The

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