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# Modalities in Vector Logic

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**Abstract** Vector logic is a mathematical model of logic in which the truth values are mapped on elements of a vector space. The binary logical functions are performed by rectangular matrices operating on the Kronecker product of their vectorial arguments. The binary operators acting on vectors representing ambiguous (fuzzy) truth values, generate many-valued logics. In this article we show that, within the formalism of vector logic, it becomes possible to obtain truth-functional definitions of the modalities "possibility" and "necessity". These definitions are based on the matrix operators that represent disjunction and conjunction respectively, and each modality emerges by means of an iterative process. The construction of these modal operators was inspired in Tarski's truth-functional definition of possibility for the 3-valued logic of Łukasiewicz. The classical Aristotelian link between possibility and necessity becomes, in the basic vector logic, a corollary of the De Morgan's connection between disjunction and conjunction. Finally, we describe extensional versions of the existential and universal quantifiers for vector logics.

***1 Introduction*** The mathematical representations of logic have opened illuminating perspectives for the understanding of the logical constructions created by the humans. These representations have also provided us with powerful technical instruments that present a wide spectrum of applications. A profound analysis of the relations between mathematics and logic can be found in Curry [4].

Recently, an algebraic representation of the propositional calculus in which the truth values are mapped on the elements of a vector space has been described. In this representation, the logical computations are performed by matrix operators. In particular, binary operations are executed by rectangular matrices that act over the Kronecker product of their vectorial arguments. This algebraic model of logic has been denominated "vector logic", and has been discovered investigating a neural model (see Mizraji [11] and [12]).

An interesting property of this formalism is the following: once a binary vector logic isomorphic in the binary domain with the classical propositional calculus is defined, the binary operators can compute ambiguous truth values,

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