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Extending Intuitionistic Linear Logic with Knotted Structural Rules

R. HORI, H. ONO, and H. SCHELLINX

Abstract In the present paper, extensions of the intuitionistic linear logic with knotted structural rules are discussed. Each knotted structural rule is a rule of inference in sequent calculi of the form: from $\Gamma, A, \ldots, A$ (n times) $\to C$ infer $\Gamma, A, \ldots, A$ (k times) $\to C$, which is called the $(n \rightsquigarrow k)$-rule. It is a restricted form of the weakening rule when $n < k$, and of the contraction rule when $n > k$. Our aim is to explore how they behave like (or unlike) the weakening and contraction rules, from both syntactic and semantic point of view. It is shown that when either $n = 1$ or $k = 1$, strong similarities hold between logics with the $(n \rightsquigarrow k)$ rule and logics with the weakening or the contraction rule, as for the cut elimination theorems, decidability and undecidability results and the finite model property.

1 Introduction In the present paper, we will introduce a new kind of structural rule, called *knotted structural rules*, and study syntactic and semantical properties of extensions of the intuitionistic linear logic with knotted structural rules. Each knotted structural rule is a rule of inference in sequent calculi of the form:

from $\Gamma, A, \ldots, A$ (n times) $\to C$ *infer* $\Gamma, A, \ldots, A$ (k times) $\to C$,

which is called the $(n \rightsquigarrow k)$-rule. It is a restricted form of the weakening rule when $n < k$, and of the contraction rule when $n > k$. Our aim is to explore how they behave like (or unlike) the weakening and contraction rules. It will be shown that when either $n = 1$ or $k = 1$, strong similarities hold between logics with the $(n \rightsquigarrow k)$ rule and logics with the weakening or the contraction rule. Therefore we can get the cut elimination theorems, decidability and undecidability results and the finite model property for them. On the other hand, we are faced with great difficulties in the remaining cases, which in the present paper we were not yet able to overcome.

In the next section, we will introduce our basic systems $\mathbf{FL_e}$ which is a sequent calculus for the intuitionistic linear logic as introduced by Girard [4], and

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