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A Revision-Theoretic Analysis of the Arithmetical Hierarchy

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Abstract In this paper we apply the idea of *Revision Rules*, originally developed within the framework of the theory of truth and later extended to a *general* mode of definition, to the analysis of the arithmetical hierarchy. This is also intended as an example of how ideas and tools from philosophical logic can provide a different perspective on mathematically more "respectable" entities. Revision Rules were first introduced by Gupta and Belnap as tools in the theory of truth, and they have been further developed to provide the foundations for a general theory of (possibly circular) definitions. Revision Rules are non-monotonic inductive operators that are iterated into the transfinite beginning with some given "bootstrapper" or "initial guess." Since their iteration need not give rise to an *increasing* sequence, Revision Rules require a particular kind of operation of "passage to the limit", which is a variation on the idea of the inferior limit of a sequence. We then define a sequence of sets of strictly increasing arithmetical complexity, and provide a representation of these sets by means of an operator $G(x, \phi)$ whose "revision" is carried out over ω^2 beginning with any total function satisfying certain relatively simple conditions. Even this relatively simple constraint is later lifted, in a theorem whose proof is due to Anil Gupta.

In this paper we apply the idea of *Revision Rules*, originally developed within the framework of the theory of truth and later extended to a *general* mode of definition, to the analysis of the arithmetical hierarchy. This is also intended as an example of how ideas and tools from philosophical logic can provide a different perspective on mathematically more "respectable" entities.

Revision Rules were first introduced in Gupta [2] and Belnap [1] and, independently, Herzberger [4] as tools in the theory of truth, and have found their most detailed exposition to date in Gupta and Belnap [3], where they provide the foundations for a general theory of (possibly circular) definitions. Revision Rules are non-monotonic inductive operators that are iterated into the transfinite begin-

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