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SINGLE AXIOM SCHEMATA FOR D AND S

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Given below* are the axiom schemata **D** and **S**, which are each sufficient for a complete propositional calculus based on a Sheffer functor,* with the appropriate rule of detachment, **I** or **I***. The schemata were inspired by the axiom of Łukasiewicz [2] for a propositional calculus with variables for propositional functions. But here we axiomatize precisely propositional calculus, not the slightly larger system of Łukasiewicz, and therefore have no need for a rule of substitution for functional variables. Further, it should be remarked that although the correspondence between the two approaches is close, such an axiomatization as that of Meredith [3], viz., $C\delta\delta O\delta p$, does not admit of an easy interpretation as an axiom schema.

Łukasiewicz' axiom is essentially a "law of bivalency": $C\delta COOC\delta O\delta p$. We will modify this to adapt it to our purposes, to the forms:

D $D(\alpha:\mathfrak{a}/\beta)DD(\alpha:\mathfrak{a}/D\beta\beta)D(\alpha:\mathfrak{a}/\gamma)(\alpha:\mathfrak{a}/\gamma)D(\alpha:\mathfrak{a}/D\beta\beta)D(\alpha:\mathfrak{a}/\gamma)(\alpha:\mathfrak{a}/\gamma)$

S $SSS(\alpha:\mathfrak{a}/\beta)(\alpha:\mathfrak{a}/\beta)SSS(\alpha:\mathfrak{a}/S\beta\beta)(\alpha:\mathfrak{a}/S\beta\beta)(\alpha:\mathfrak{a}/\gamma)SS(\alpha:\mathfrak{a}/S\beta\beta)$
$(\alpha:\mathfrak{a}/S\beta\beta)(\alpha:\mathfrak{a}/\gamma)SS(\alpha:\mathfrak{a}/\beta)(\alpha:\mathfrak{a}/\beta)SSS(\alpha:\mathfrak{a}/S\beta\beta)(\alpha:\mathfrak{a}/S\beta\beta)$
$(\alpha:\mathfrak{a}/\gamma)SS(\alpha:\mathfrak{a}/S\beta\beta)(\alpha:\mathfrak{a}/S\beta\beta)(\alpha:\mathfrak{a}/\gamma)$

And our rules of detachment are to be the relatively weak rules

$$\mathbf{I}\quad \frac{D\alpha D\beta\beta \qquad \alpha}{\beta} \qquad\qquad \mathbf{I}^*\quad \frac{SSS\alpha\alpha\beta SS\alpha\alpha\beta \qquad \alpha}{\beta}$$

In our axiom schemata, and throughout this paper, we use the following conventions: Lower case Greek letters are variables for well formed formulas; lower case German letters are variables for the propositional variables (which are lower case Latin letters); an expression of the kind $\alpha:\mathfrak{a}/\beta$ means the formula resulting from the formula α by substitution of the formula β for every occurrence of the variable $\mathfrak{a}$ in α; an asterisk indicates replacement of D by S throughout the formula in question.

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*The terminology here used is largely explained in [4]. The suggestion that these axiomatizations be treated as schemata is due to Prof. Sobociński.

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