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A SET OF AXIOMS FOR THE PROPOSITIONAL CALCULUS WITH IMPLICATION AND NON-EQUIVALENCE

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It is well-known that implication and non-equivalence constitute a complete system of independent primitive connectives for the propositional calculus. In this article it is the intention of the author to give an independent set of axioms by means of the two connectives mentioned above, the rules of inference being substitution and *modus ponens*.

In §1 we state the axioms and prove some preliminary theorems. In §2 we solve the decision problem. Finally, we establish the independence of the axioms and rules in §3. In the matter of notation we shall follow Alonzo Church[1].

§1. *Axioms and Preliminary Theorems*. The axioms of our logistic system, say P, are the seven following:

Axiom 1. $p \supset \cdot q \supset p$

Axiom 2. $s \supset [p \supset q] \supset \cdot s \supset p \supset \cdot s \supset q$

Axiom 3. $p \supset q \supset p \supset p$

Axiom 4. $p \supset [p \not\equiv q] \supset \cdot q \supset \cdot p \not\equiv q$

Axiom 5. $p \not\equiv q \supset \cdot p \supset q \supset q$

Axiom 6. $p \not\equiv q \supset \cdot p \supset \cdot q \supset s$

Axiom 7. $p \not\equiv q \supset \cdot q \not\equiv p$

In fact, as is evident from the above set, any formulation of the implicational propositional calculus and Axioms 4-7 will suffice. We note that from the present formulation the deduction theorem—to be henceforth referred to as D.T.—follows immediately.

We now go on to prove some theorems.

1. Church, A. *Introduction to Mathematical Logic*, I. Princeton, N. J., 1956.

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