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TOPOLOGICAL GEOMETRIES AND A NEW CHARACTERIZATION OF $\mathbf{R}^m$

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INTRODUCTION*. Mathematicians engaged in research in one area of study may find concepts which have already been defined and studied in another area, and they can therefore draw on those definitions and results which already exist. In this way, topology has already been used to study geometry; for example, in certain axiomatizations of Euclidean plane geometry, "natural" topologies, having as subbasis elements either half-planes or interiors of triangles, can be defined on the underlying set, and these topologies can then be used in the formulation of propositions, or further axioms.

Starting with chapter II in this paper, however, the underlying set on which a "geometry" is defined is assumed already to possess the structure of a topological space. The geometry is an additional structure with axioms to bind topological and geometric structures together. We may compare what has been done to the example of a topological group, where two structures, algebraic and topological, are related by the continuity of the group operation.

The usual manner by which a structure is given to a point set X is by the selection of certain distinguished subsets either of X itself, or of certain sets related to X, or of both. A topology on X is defined using a family of distinguished subsets of X, while an operation on X is defined by a subset of $(X \times X) \times X$. Classical geometries usually call for, either implicitly or explicitly, the existence of distinguished subsets called lines, planes, or k-dimensional subspaces. We define a geometry on a set X in terms of distinguished subsets of X called k-flats, which are generalizations of k-dimensional subspaces.

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