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SYSTEMS CLASSICALLY AXIOMATIZED AND PROPERLY CONTAINED IN LEWIS'S S3

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It is well known that Lewis's modal systems S3, S4 and S5 can be classically axiomatized. That is, an axiomatic for those systems can be given with a finite number of axioms taking substitution for propositional variables and material detachment as the only primitive rules of inference. It will be shown in this paper that such an axiomatic is available for some systems properly contained in S3. Each section of the paper introduces new axiomatics for sub-systems of S3 and then gives new sub-systems which are classically axiomatized and in which all of Lewis's primitive rules of inference are derivable. The symbolism throughout is that of [7] and "α is a thesis" is abbreviated as "$\vdash\alpha$".

1. Lemmon in [4] gave new foundations for Lewis's systems S1-S3 of [5] analogous to a systematic for T of Feys-von Wright [2, 8, 12] due to Gödel in [3]. In this section new foundations for Lemmon's systems are described and two systems containing Lemmon's S0.5 and properly contained in S3 are classically axiomatized.

The Lemmon systems are N-C-L calculi with K and E defined in the usual way by C and N, $\mathfrak{C}$ defined as LC and $\mathfrak{E}pq$ (strict equivalence) as $K\mathfrak{C}pq\mathfrak{C}qp$. Propositional calculus (PC) is given by three rules:

(PCa) if α is a tautology, then $\vdash\alpha$;
(PCb) substitution for propositional variables;
(PCc) material detachment (that is, from α and $C\alpha\beta$ infer β);

and Lewis's systems are based on selections from the following rules and axioms:

(a) $\vdash\alpha$ only if $\vdash L\alpha$;
(a') α is a tautology or axiom only if $\vdash L\alpha$;
(a'') α is a tautology only if $\vdash L\alpha$;
(b) $\vdash LC\alpha\beta$ only if $\vdash LCL\alpha L\beta$;
(b') substitutability of strict equivalents;
(b'') $\vdash\mathfrak{E}\alpha\beta$ only if $\vdash\mathfrak{E}L\alpha L\beta$;
(1) $CLCpqLCLpLq$;
(1') $CLCpqCLpLq$;
(2) $CLpp$;
(3) $CKLCpqLCqrLCpr$.

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