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STRONG REDUCIBILITY ON HYPERSIMPLE SETS[1]

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1. *Introduction.* In [5], Yates noted that all those hypersimple sets constructed by Dekker in [2] are nonhyperhypersimple sets with retraceable complements. The main purpose of this note is to draw attention to an easy construction, modifying very slightly the usual "cylinder" mapping, which (in view of the second theorem of [4]) leads to a proof that there are hypersimple, nonhyperhypersimple sets whose complements are not regressive in the sense of [3]. Actually, the mere existence of sets meeting this description is very easily shown on the basis of propositions in [3], [4], and [5]; what we wish to emphasize is that our construction provides an effective procedure for passing from a given hyperhypersimple set β to a hypersimple, nonhyperhypersimple, noncoregressive set α such that $\alpha \equiv_m \beta$.

For convenience, we use the following abbreviations: '**HS**', for the class of hypersimple sets; '**HHS**', for the class of hyperhypersimple sets; '$\bar{\mathbf{R}}$', for the class of sets with regressive complement. We denote the set of all natural numbers by '**N**'.

2. *Immune Cylindrification.* Let $\tau(x,y)$ be the usual recursive pairing function:

$$\tau(x,y) = x + \frac{1}{2}(x+y)(x+y+1);$$

let $\tau_1(x)$, $\tau_2(x)$ be its associated "unpairing" functions... thus, $x = \tau(\tau_1(x), \tau_2(x))$, for all x. We use the words "isolated set" as is customary: the number set β is isolated iff β is either finite or immune.

Lemma 1. Let $\{\omega_{\xi(n)}\}$ be a recursive sequence of r.e. sets (thus, the indexing function ξ is assumed recursive). Let α be an immune set. Let $\beta =_{df} \tau(\{0\} \otimes \alpha) \cup \bigcup_{n>0} \tau(\{n\} \otimes (\alpha - \omega_{\xi(n-1)}))$. Then β is immune $\Leftrightarrow$ $(\forall k \epsilon \alpha)$ $(\{y | k \notin \omega_{\xi(y)}\}$ is isolated).

(We use '$\otimes$' to denote the Cartesian product operation.)

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