

A NOTE ON THE AXIOMATIZATION OF RUBIN'S SYSTEM (S)

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Rubin, in [3], suggests that the axiomatic for system (S) may be simplified. It is here shown that

R5 If $\vdash \alpha$ in (S) then $\vdash \Box_2 \alpha$ in (S)

and

R*5 If $\vdash \alpha$ in (S) then $\vdash \Box_1 \alpha$ in (S)

are derivable from the other axioms and rules of (S).

This paper presupposes [3] and adopts the same primitive basis and definitions for (S). Thus the axioms and rules of (S) are

A1 $(\alpha \wedge \beta) \Rightarrow (\beta \wedge \alpha)$

A2 $(\alpha \wedge \beta) \Rightarrow \alpha$

A3 $\alpha \Rightarrow (\alpha \wedge \alpha)$

A4 $((\alpha \wedge \beta) \wedge \gamma) \Rightarrow (\alpha \wedge (\beta \wedge \gamma))$

A5 $\alpha \Rightarrow \sim \sim \alpha$

A6 $((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \gamma)) \Rightarrow (\alpha \Rightarrow \gamma)$

A7 $(\alpha \wedge (\alpha \Rightarrow \beta)) \Rightarrow \beta$

A8 $\Box_2 \alpha \Rightarrow \Box_2 \Box_2 \alpha$

A12 $\Box_2 \alpha \Rightarrow \Box_1 \alpha$

R1 If $\vdash \alpha$ and $\vdash (\alpha \Rightarrow \beta)$ then $\vdash \beta$.

R2 If $\vdash \alpha$ and $\vdash \beta$ then $\vdash (\alpha \wedge \beta)$.

R3 If $\vdash (\alpha \Leftrightarrow \beta)$ and $\vdash \gamma$ and δ results from γ by replacing α for β (or β for α) then $\vdash \delta$.

together with **A*1-A*8**, **R*1-R*3**. (T^* is the wff obtained from T by replacing all the " \Box_2 's" by " \Box_1 's".)

The following theorems, **S1-S7**, follow from **A1-A8** and **R1-R3**, and their proofs can be found in [1].¹

S1 $\alpha \Rightarrow \alpha$ [12.1]

S2 $((\alpha \supset \beta) \wedge (\beta \supset \gamma)) \Rightarrow (\alpha \supset \gamma)$ [15.1]

S3 $\Box_2 \alpha \Rightarrow \alpha$ [18.42]

S4 $(\alpha \Rightarrow \beta) \Leftrightarrow \Box_2(\alpha \supset \beta)$ [18.7]