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A NOTE ON CONTINUOUS GAMES, THE NOTION OF STRATEGY AND ZERMELO'S AXIOM*

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1. Introduction. The notion of a continuous game has appeared very naturally from a generalization of discrete (or sequential) games, and partly stems from the simple fact that a time scale is more often than not considered continuous. While for sequential games, finitist as they are, (see M. Claude Berge), the notion of strategy does not bring set-theorectical difficulties, the same is not the case for continuous games. In fact, the problem of existence of a single (pure) strategy for such games is strongly linked to Zermelo's Axiom of Choice for infinite sets.

A particular approach to continuous games has, in the recent years and because of its immediate practical applications, received increased attention. It was started by Rufus Isaacs in 1954 and 1955 in a series of Rand Corporation memoranda which were followed by, among others, papers by Scarf, Fleming and Berkovitz, and goes under the general heading of "Differential Games". The method is akin to and includes the widespread one in Control Theory, and is based on a set of differential equations:

$$\dot{x} = G(x : \phi; \psi; \ldots .)$$

where x is the state vector and $\phi; \psi; \ldots .$ are vector-valued functions of time chosen independently by the various players; associated with one or more optimization criteria (integral or terminal pay-offs) etc. .

Intrigued by the implications of the existence of pure strategy theorems for such games, the author has devised a very general set-theoretical description of a n-person continuous game with simultaneous moves (that is, a description of the game in extensive form) and attempted to determine the power of the sets involved. In behalf of clarity, we present here a two-person simplified game only. It seems that the same reasoning could be applied to the question of existence of single solutions to sets of differential equations without mentioning game theory, mutatis mutandis.

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