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ON THE CONNECTION OF THE FIRST-ORDER FUNCTIONAL CALCULUS WITH $\aleph_0$ PROPOSITIONAL CALCULUS

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A simply conclusion from papers [2]-[5] is that for each formula E we may construct a $n(E)$-valued propositional calculus such that if E is not a thesis, then E is false in this calculus by a finite interpretation of the quantifiers; by means of a simply extending of the $n(E)$ valued calculus to $\aleph_0$ propositional calculus we may prove in one the converse theorem. This method we have used in [5] and have proved that it is possible to approximate the first-order functional calculus by many valued propositional calculi.

An interest approximation of the first-order functional calculus by $\aleph_0$ propositional calculus follows from [3] and [4]. We obtain it by means of constructing of a correspondence between atomic formulas and sequences of numbers *0* and *1* such that:

1. If the atomic formula is of ≥ 2 arguments, then the correspondents sequence is periodic/we shall give the period/.
2. The difference in this correspondence is in general on atomic formulas of one argument whose we must consider an infinite number.
3. For some formulas, e.g. $\Sigma a_1\ \Sigma a_2\ \Pi a_3 \ldots\ \Pi a_k F$ where F is quantifier and individual variable—free, monadic formulas, . . . , the $\aleph_0$ calculus may be replaced by suitable n- or *2*-valued propositional calculus; one follows from a general theorem.

We shall use the notation of all mentioned papers and in particular:

(1) variables: (1°) individual: $x_1, x_2, \ldots$ /or simply x/, (2°) apparent: $a_1, a_2, \ldots$ /or simply a/,

(2) finite numbers of functional variables: $f_1^1, \ldots, f_q^1, f_1^2, \ldots, f_q^2, \ldots, f_1^t, \ldots, f_q^t$ / f_i^m of m-arguments, $m = 1, \ldots, t$ and $i = 1, \ldots q$/

(3) logical constants: (negation), + (alternative), Π(general quantifier),

(4) atomic expression: $R, R_1, R_2, \ldots$; expressions: $E, F, G, E_1, F_1, G_1 \ldots$ [1]

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1. Expressions and formulas we define in the usual way; the expression in which an apparent variable a belong to the scope of two quantifiers Πa is not a formula; if a does not occur in E, then ΠaE is not a formula.

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