Notre Dame Journal of Formal Logic
Volume V, Number 4, October 1964

FAMILY *K* OF THE NON-LEWIS MODAL SYSTEMS

BOLESŁAW SOBOCIŃSKI

In [6], p. 109, a regular modal formula α is defined as a modal formula which after deleting the modal functors L and M, if they occur in α, and after replacing the modal functors for more than one argument, if they occur in α, by the corresponding classical functors, throughout α, this formula becomes a thesis of the bi-valued propositional calculus. If a regular modal formula α is such that its addition as a new axiom to S5 reduces such extension of the latter system to the classical propositional calculus, I say that α is non-Lewis modal formula. Correspondingly, the modal systems which are irreducible to the classical propositional calculus and which are obtained by the addition of one or more non-Lewis modal formulas to the proper subsystems of S5 will be called here the non-Lewis modal systems. Such a system, for example, is constracted by McKinsey, *cf.* [2], by adding to S4 the new axiom

K1 *ℭKLMpMLqMKpq*

which, clearly, is non-Lewis modal formula. As I have proved in [7], pp. 77-78, in this system, which is called by McKinsey S4.1, but which I call more conveniently system K1, axiom *K1* can be substituted equivalently by several other formulas, as, e.g., by

K2 *ℭLMpMLp*

or by

K4 *LMLCpLp*

This fact will be used later.

In this paper I shall present some investigations, which are far from being complete, concerning certain family *K* of the non-Lewis modal systems. I define this family *K* as a class of such and only such modal systems that each of them satisfies the following three conditions:

1) it is a proper normal extension of S4,
2) it is irreducible to the classical propositional calculus,
3) it contains as its axiom or its consequence formula *K2*.

Received September 29, 1964