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K1, K2 AND RELATED MODAL SYSTEMS.

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1. Sobociński refers in [5] to two systems which he calls K1 and K2. If S4 is axiomatised with the rule to infer $\vdash L\alpha$, from $\vdash\alpha$, these systems are axiomatisable by adding *CLMpMLp* and *ELMpMLp* respectively to S4. It is obvious that K1 is a subsystem of K2, since *ELMpMLp* is equivalent to *CLMpMLp* plus its converse *CMLpLMp*; Sobociński, in conclusion, raises the question whether it is a "proper" subsystem. This question is equivalent to the question whether, given S4, *CMLpLMp* is independent of *CLMpMLp*. That it is, may be established by the following matrix:—

	C	1	2	3	4	5	6	7	8	N	M	L
*	1	1	2	3	4	5	6	7	8	8	1	1
	2	1	1	3	3	5	5	7	7	7	2	6
	3	1	2	1	2	5	6	5	6	6	3	7
	4	1	1	1	1	5	5	5	5	5	4	8
	5	1	2	3	4	1	2	3	4	4	1	5
	6	1	1	3	3	1	1	3	3	3	2	6
	7	1	2	1	2	1	2	1	2	2	3	7
	8	1	1	1	1	1	1	1	1	1	8	8

This verifies S4 and *CLMpMLp*, but falsifies *CMLpLMp* when $p = 2, 3, 6$ or 7.

The history of this matrix is worth giving, as it suggests solutions to certain connected problems.

2. In [3], [4] and other papers an interpretation is given for modal functors which may be re-stated, more in the spirit of [2], as follows:— Use *p*, *q*, *r*, etc. for propositional variables and *a*, *b*, *c*, etc. for "worlds" or total states of affairs. Let *U* represent a certain relation between worlds, and write *Tap* for "It is the case in world *a* that *p*". Assume, beside quantification theory and identity theory, the following:—

1. *ETANpNTap*
2. *ETaCpqCTapTaq*
3. *ETaLpΠbCUabTbp*

From these, given *Mp* as short for *NLNp*, it is easy to deduce

4. *ETaMpΣbKUabTbp*

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