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ON THE PROPOSITIONAL SYSTEM A OF VUČKOVIĆ AND ITS EXTENSION. II

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6.* Completeness of $\mathcal{A}$. The axioms *F1-F18* given in **5**, together with the rules of procedure **RI** and **RII** are verified by the matrices $\mathfrak{M}1$-$\mathfrak{M}4$. Therefore, in order to prove that system $\mathcal{A}$ determined by these matrices is finitely asiomatizable it has to be shown that every thesis verified by $\mathfrak{M}1$-$\mathfrak{M}4$ is a consequence of the axioms *F1-F18* taken together with the rules **RI** and **RII**. Such a proof can be obtained in several ways, and here I shall present the following one:

Let us assume that there are the theses verified by $\mathfrak{M}1$-$\mathfrak{M}4$ and which are independent from the adopted axiom-system *F1-F18*. Hence, among them there must exist the shortest independent thesis. It will be shown that such a thesis does not exist, and, therefore, that every thesis verified by $\mathfrak{M}1$-$\mathfrak{M}4$ is a consequence of *F1-F18* taken together with **RI** and **RII**.

6.1 This proof will be conducted as follows. Let us assume that there exists formula $\mathfrak{A}$ which is the shortest independent thesis. Then, it possesses a certain structural form, i.e. it belongs to a certain structural type **T**. Hence:

(i) If in the field of $\mathcal{A}$ every formula $\mathfrak{B}$ belonging to the given type **T** is inferentially equivalent to one or several such formulas that each of them either is shorter than $\mathfrak{B}$ or is a consequence of *F1-F18* or is falsified by $\mathfrak{M}1$-$\mathfrak{M}4$, then, obviously, the shortest independent thesis $\mathfrak{A}$ cannot belong to the type **T**.

(ii) On the other hand, if in the field of $\mathcal{A}$ every formula $\mathfrak{B}$ belonging to the given type **T** is inferentially equivalent to one or several such formulas that 1) at least one of these formulas belongs to certain type **T'** which is simpler in some respect than **T**, and 2) the remaining formulas are shorter than $\mathfrak{B}$, then, obviously, in the field of $\mathcal{A}$, $\mathfrak{B}$ is a consequence of the independent

*The first part of this paper appeared in *Notre Dame Journal of Formal Logic*, v. V (1964), pp. 141-153. It will be referred throughout this part as [14]. See the additional Bibliography given at the end of this part. An acquaintance with [14] is presupposed.

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