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AN AXIOMATIZATION OF PRIOR'S MODAL CALCULUS Q

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Prior defines a model for a modal calculus **Q** (cf [1], pp. 43f):

The truth values are infinite sequences of *1*'s, *2*'s, and *3*'s, with the proviso that the first term of each sequence is not *2*. The designated values are those with no *3*'s.

The values of propositional operators are found by applying the tables

Ka_ib_i	1	2	3 (b_i)
1	1	2	3
2	2	2	2
(a_i) 3	3	2	3

Na_i	
1	3
2	2
(a_i) 3	1

to the terms of the sequences $<a_1, a_2, a_3, \ldots>$ and $< b_1, b_2, b_3, \ldots >$. The other propositional operators can be defined from these in the usual way.

For formal convenience I shall use L for what is NMN in Prior's system, and $\mathbf{L}$ for his L. These operators are given by

$$\mathbf{L}\alpha \text{ is } \begin{cases} <1,1,1,\ldots> \text{ when } \alpha \text{ is } <1,1,1,\ldots>; \\ 2 \text{ where } \alpha \text{ is } 2 \text{ and } 3 \text{ elsewhere, when } \alpha \text{ consists of } 1\text{'s and } 2\text{'s}; \\ 2 \text{ where } \alpha \text{ is } 2 \text{ and } 3 \text{ elsewhere, when } \alpha \text{ has a } 3. \end{cases}$$

$$L\alpha \text{ is } \begin{cases} <1,1,1,\ldots> \text{ when } \alpha \text{ is } <1,1,1,\ldots>; \\ 2 \text{ where } \alpha \text{ is } 2 \text{ and } 1 \text{ elsewhere, when } \alpha \text{ consists of } 1\text{'s and } 2\text{'s}; \\ 2 \text{ where } \alpha \text{ is } 2 \text{ and } 3 \text{ elsewhere, when } \alpha \text{ has a } 3. \end{cases}$$

This paper is devoted to showing that **Q** can be axiomatized by adding to **PC** the following axioms and rules:

1 $CLpp$
2 $C\mathbf{L}pp$
3 $CK\mathbf{L}p\mathbf{L}q\mathbf{L}Kpq$
RQ$L\alpha$ $C\beta\gamma \implies C\beta L\gamma$,

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