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A NATURAL DEDUCTION SYSTEM FOR MODAL LOGIC

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This paper relates a particular system of propositional calculus (hereafter referred to as **F** and described in §1 below), suggested by Dr. Milton Fisk during the seminar in symbolic logic at the University of Notre Dame, to the Lewis modal logic **S4** [4]. **F** has no axioms--its description begins by laying down certain rules as basic and it proceeds by inferring rules from its basic rules. Thus **F** may be considered as a systematic for rules which govern formulas, where Lewis's system is considered as a systematic for formulas. Indeed, if the basic rules of **F** are interpreted as claiming that certain forms of arguments are valid, for instance, if *F1* is taken to mean that any argument of the form "α, β; therefore $(\alpha \wedge \beta)$" is valid, then the basic rules can accurately be called principles of (propositional) logic. As so interpreted, *F1*-*F7* provide a basis for systematizing logical principles. **F** then becomes a systematic for evaluating individual arguments: an argument is valid if it is governed by a principle which can be derived in **F**.

In this paper a system **A** is said to *imply inferentially* a system **B** if and only if the axioms and rules of **B** stated in the primitive notation of **B** can be *inferred* in **A**. Thus §2 shows that **F** inferentially implies **S4.** But **S4** does not inferentially imply **F** (and hence **F** and **S4** are not inferentially equivalent), since the rules of **F** cannot be inferred in **S4**--they hold for wffs while those of **S4** hold only for theses. But since §3 contains a formal proof that every thesis of **F** is a thesis of **S4** and since every thesis of **S4** is a thesis of **F** (as a corollary of §2), it is shown that the two systems are formally equivalent in the sense that they have the same set of theses.

The description of system **F** that appears here differs from that description of the systematic for arguments which Dr. Fisk originally suggested, in that the metarule of replacement (**FII**) which he had taken as basic is derived from the basic rules.

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