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THE PRAGMATICS OF TRUTH FUNCTIONS

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A sentential calculus may be conceived as a pair $\langle \mathbf{S}, \mathbf{R} \rangle$, where $\mathbf{S}$ is a set of sentences generated by the operations of infixing "." and prefixing "$\sim$" from a given non-empty set of unanalyzed sentences, and $\mathbf{R}$ is the smallest equivalence relation on $\mathbf{S}$ which meets the following conditions:

1. $\mathbf{R}(s \cdot s', s' \cdot s)$,
2. $\mathbf{R}(s \cdot (s' \cdot s''), (s \cdot s') \cdot s'')$,
3. $\mathbf{R}(s \cdot \sim s', s'' \cdot \sim s'')$ *if and only if* $\mathbf{R}(s \cdot s', s)$,
4. *if* $\mathbf{R}(s, s')$, *then* $\mathbf{R}(s \cdot s'', s' \cdot s'')$,
5. *if* $\mathbf{R}(s, s')$, *then* $\mathbf{R}(\sim s \cdot \sim s')$,

for all s, s', s'' in $\mathbf{S}$. If an extra-logical axiom s_0 is adjoined to the sentential calculus, then $\mathbf{R}$ is the smallest equivalence relation on $\mathbf{S}$ which meets 1 - 5 and:

6. $\mathbf{R}(s_0, \sim(s \cdot \sim s))$,

for some $s \varepsilon \mathbf{S}$. If there exist at least two equivalence classes, then $\mathbf{B} = \mathbf{S}/\mathbf{R}$, the set of equivalence classes of $\mathbf{S}$ under $\mathbf{R}$, has the structure of a non-trivial Boolean algebra. If $p, q \varepsilon \mathbf{B}$ and $s \varepsilon p$, $s' \varepsilon q$, then $p \wedge q$ and $\bar{p}$ may be defined respectively as the equivalence classes of $s \cdot s'$ and $\sim s$. The ordered triple $\langle \mathbf{B}, \wedge, - \rangle$ may be seen to be a Boolean algebra.

A Boolean logic, or logic of truth functions, may be conceived as a Boolean algebra $\langle \mathbf{B}, \wedge, - \rangle$ together with a sum ideal (or filter) $\mathbf{I}$ of $\mathbf{B}$. For in the case that $\mathbf{I}$ is a maximal proper sum ideal, the induced algebra on $\mathbf{B}/\mathbf{I}$ is simple and may be considered as the Boolean algebra of truth values. Hence the homomorphism from $\langle \mathbf{B}, \wedge, - \rangle$ to the induced algebra on $\mathbf{B}/\mathbf{I}$ may be considered as an interpretation of the elements of $\mathbf{I}$ as true (in the present case, the proposition containing sentences equivalent to some tautology), and of the remaining elements of $\mathbf{B}$ as false. Thus where $\mathbf{I}$ is maximal and proper, we may say that $\langle \mathbf{B}, \mathbf{I}, \wedge, - \rangle$ is complete and consistent.

These considerations suggest that if a set $\mathbf{S}$ of sentences, such as is described above, is given together with a set of performances of the users of $\mathbf{S}$, then one might find a characterization of subsets of performances which will induce on $\mathbf{S}$ the structure appropriate for a truth-functional logic.

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