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THREE AXIOM NEGATION-ALTERNATION FORMULATIONS OF THE TRUTH-FUNCTIONAL CALCULUS

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There are numerous formulations of the calculus of truth functions (so-called propositional logic) of the traditional axioms-plus-rules-of-derivations type in current use. Yet each has a combination of features of its own which gives it certain advantages for particular purposes, if only didactical, or simply makes it the favorite of some logicians as a matter of personal taste. It may still be of service, therefore, to record some more such systems which might be profitably used. The purpose of this communication is to note the existence of very simple such logistic formulations of the truth-functional calculus in which alternation is the primitive binary connective, but which, unlike the familiar Hilbert-Ackermann system, have only three axioms (or axiom-schemata).

There are several negation-alternation primitive bases for the truth-functional calculus with three or fewer axioms already recorded in print, but they are hardly, if ever used; apparently, it is felt that objectionable features in them make the reduction of the number of axioms from the Hilbert-Ackermann system, or the retention of alternation as the primitive binary connective, for whatever merit is seen in it, not worthwhile.[1] A diligent search has failed to reveal that any of the systems to be presented here have been proposed before; they are all very similar to each other, and we will hence treat one, which we will refer to as the system Σ_0, as basic and consider the others as variations of it.

We will use familiar vocabulary and formation rules for the object language, with '$\sim$' and '$\vee$' as our primitive connectives; for the abbreviation of wffs, besides the omission of parentheses, we will have occasion to employ only '$\supset$' as a defined connective in the usual manner.

As is the case for all such logistic systems, there are of course two versions of Σ_0 (and of each of its variations), namely with a finite or an infinite axiom set respectively. For the purpose of this presentation we adopt a finite axiom set—there is no intent thereby to express a preference for this approach over the one using axiom schemata in all contexts. Our rules of derivation then are the usual *substitution* and *modus ponens*. We will, of

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