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REMARKS ABOUT AXIOMATIZATIONS OF CERTAIN MODAL SYSTEMS

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In this paper I present some remarks about axiomatizations of certain modal systems investigated by several authors. Mostly, it will be shown that the axiom--systems of theories under consideration can be simplified. I shall use here a modification of Łukasiewicz's symbolism in which "C", "K", "A", and "N" possess the ordinary meaning and "M", "L", "$\mathfrak{C}$" and "$\mathfrak{E}$" mean "$\Diamond$", "$\sim\Diamond\sim$", "$-\!\!3$" and "$=$" respectively. Symbol "$\vdash\alpha$" means always: formula α is provable in the system under consideration. If it will be not stated clearly to the contrary, it is always assumed tacitly that a system under consideration has Lewis' primitive terms and rules of procedure. An acquaintance with the modal systems of Lewis is presupposed. The systems often mentioned below, S1° - S4°, are defined in [3] and [9], pp. 52-53.

1. An elementary lemma presented below is used several times in this paper. Consider the following two sets, **V** and **W**, of formulas and meta-rules.

V

V1 $\mathfrak{C}LKpqKLpLq$
V2 $\mathfrak{C}KLpLqLKpq$
V3 $\mathfrak{C}CpqCNqNp$
V4 $\mathfrak{C}CLpMqNKLpLNq$
V5 $\mathfrak{C}NLKpNqMCpq$
V6 $\mathfrak{C}NKLpLNqMCpq$
V7 $\mathfrak{C}MCpqNLKpNq$
V8 *If* $\vdash\mathfrak{C}\alpha\beta$ *and* $\vdash\mathfrak{C}\beta\gamma$, *then* $\vdash\mathfrak{C}\alpha\gamma$
V9 *If* $\vdash\alpha$ *and* $\vdash\mathfrak{C}\alpha\beta$, *then* $\vdash\beta$
V10 The rule of substitution ordinarily used in modal systems

W

W1 $CLKpqLKpLq$
W2 $CKLpLqLKpq$
W3 $CCpqCNqNp$
W4 $CCLpMqNKLpLNq$
W5 $CNLKpNqMCpq$
W6 $CNKLpLNqCLpMq$
W7 $CMCpqNLKpNq$
W8 $CCpqCCqrCpr$
W9 *If* $\vdash\alpha$ *and* $\vdash C\alpha\beta$, *then* $\vdash\beta$
W10 The rule of substitution ordinarily used in modal systems

LEMMA 1. For any modal system T, if either every element of **V** *or every element of* **W** *is a consequence of T, then*

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