62
Notre Dame Journal of Formal Logic
Volume V, Number 1, January 1964

FURTHER AXIOMATIZATIONS OF THE ŁUKASIEWICZ THREE-VALUED CALCULUS

FEDERICO M. SIOSON

A propositional calculus for three-valued logic was first constructed by J. Łukasiewicz (1920) and subsequently communicated in a lecture before the Polish Philosophical Society. His results were published later [2]. In 1931 M. Wajsberg [4] formalized the three-valued logic of Łukasiewicz by means of two primitive connectives, implication (denoted by C) and negation (denoted by N), and the following axioms stated in the Łukasiewicz convention:

W_1. $CpCqp$

W_2. $CCpqCCqrCpr$

W_3. $CCNpNqCqp$

W_4. $CCCpNppp$.

Wajsberg also assumed the following rules of inference:

S. Any well-formed formula may be substituted for a propositional variable in all its occurrences in a theorem or axiom.

MP. If P and CPQ are theorems, then Q is also a theorem.

The truth tables for C and N of the Łukasiewicz three-valued logic is given by

Cpq	F	U	T	Np
F	T	T	T	T
U	U	T	T	U
T	F	U	T	F

In 1951 Alan Rose [3] introduced several new other axiomatizations of the same propositional logic by taking disjunction (denoted by A) and negation as primitives and substitution and the following as rules of inference:

MP_1. If P and $ANPQ$ are theorems, then Q is also a theorem.

Received September 8, 1963